# New Method for Determining Energies of Cosmic-Ray Nuclei 

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#### Abstract

A new procedure for determining the energies of particles of primary cosmic radiation is described. The procedure is based on measuring the spatial density of the flux of secondary particles originating from the first event of nuclear interaction that have traversed a thin-converter layer. The use of the proposed method makes it possible to create equipment of comparatively small mass and high sensitivity. The procedure can be applied in balloon- and satellite-borne cosmic-ray experiments with cosmic nuclei for all types of nuclei over a wide energy range between $10^{11}$ and $10^{16} \mathrm{eV}$ per particle. Physical foundations of the method, results of a simulation, and the applicability range are described.


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## INTRODUCTION

Investigation of primary cosmic rays has been of interest for astrophysics since the discovery of cosmic rays. Processes occurring in the Milky Way Galaxy and maybe beyond it are reflected in the chemical composition of cosmic rays, in the energy spectra of cosmic-ray components, and in their possible anisotropy. The spectrum and the composition of primary cosmic rays have been explored with the aim of obtaining answers to the fundamental questions of the origin of primary cosmic radiation, the mechanisms of their acceleration, and their propagation in the Milky Way Galaxy.

In the energy range $10^{11}-10^{16} \mathrm{eV}$, which is usually of prime interest, the energy spectrum of cosmic rays behaves as follows. For $10^{11}<E<3 \times 10^{15} \mathrm{eV}$, it can be approximated by a power-law function proportional to $E^{-\gamma}$ with $\gamma \sim 1.7$; at $E \sim 3 \times 10^{15} \mathrm{eV}$, the spectrum has a knee, becoming steeper, which is described by a value of $\gamma \sim 2.2$. There are a few different interpretations of the knee phenomenon in the spectrum of cosmic rays, but none of these has been corroborated experimentally. This is because there are no data from a direct investigation of the chemical composition of primary cosmic rays in the region $E>10^{15} \mathrm{eV}$; as to data in the energy region immediately below the knee ( $E=10^{14}-10^{15} \mathrm{eV}$ ), their statistical significance is insufficient. The knee phenomenon was discovered by means of an indirect procedure that employs extensive air showers and which makes it possible to determine, to a rather high degree of precision, the energy spectrum of the sum of all cosmic-ray components over a wide energy region ( $E>10^{15} \mathrm{eV}$ ), but which cannot pinpoint the
type of a primary particle. Results obtained by this method for the chemical composition of primary cosmic radiation are still hotly debated [1]. In order to explore the energy range $E=10^{14}-10^{16} \mathrm{eV}$, which is of crucial importance for the astrophysics of highenergy cosmic rays, it is necessary to study directly the composition of cosmic rays beyond the atmosphere, which fully transforms the primary flux. This requires deployment of large-area arrays and long exposure times.

The main difficulty in directly investigating cosmic rays over the aforementioned energy range is that, of the entire toolkit of procedures that contemporary experimental physics provides for simultaneously measuring the energies of all types of $Z=1-26 \mathrm{nu}-$ clei (this is of paramount importance for determining the relationship between the intensities of different nuclei), only two can be applied in the case being discussed. These are the magnetic-spectrometer and the ionization-calorimeter procedure. However, the potential of the first procedure is severely constrained by the need for generating magnetic fields of enormous strength beyond the atmosphere. In view of the current state of the art in superconducting technologies, such investigations into the energy range above 1 TeV will become possible only in the future. Over the past 30 years, the ionization-calorimeter procedure has been the main tool in experiments with highenergy cosmic rays. It furnished unique results in experiments like PROTON [2] and SOKOL[3] and in experiments where a modified ionization-calorimeter procedure is implemented with the aid of a facility that employs x-ray emulsion chambers (JACEE [4], MUBEE [5], RUNJOB [6]). Over the past two
decades, however, the experimental astrophysics of cosmic rays has run into considerable difficulties, since arrays having a weight of a few tons must be placed beyond the atmosphere for a long time in order extend investigations with ionization calorimeters to energies in excess of $10^{14} \mathrm{eV}$. This obviously makes such investigations extremely expensive. There are also limitations on the use of the procedure based on x-ray emulsion chambers: first, long-term exposures (of duration in excess of 250 h ) of nuclear emulsions and x-ray films is impossible; second, treatment of primary data requires painstaking efforts.

For performing investigations in the vicinity of the knee in the spectrum of primary cosmic radiation, it is crucial to create detecting equipment having a relatively low weight and a high sensitivity and providing the possibilities for long-term exposures and for explorations of cosmic rays by a single procedure over a wide energy range (of a few orders of magnitude). For this, it is necessary to develop new approaches that would make it possible to determine the energies of ultrahigh-energy particles without employing thick absorbers.

A procedure that is a development of the wellknown and extensively used method for determining the primary-particle energy from the mean angle of divergence of secondaries originating from an inelastic-interaction event (Castagnoli's method [8]) was proposed in [7] on the basis of experience gained in previous investigations. This procedure, in contrast to that based on ionization calorimetry, does not require a thick absorber of energy-a thin target of depth about a few centimeters is sufficient. In the following, we describe the physical foundations of the method and present results obtained from a simulation of it.

## 1. METHOD FOR DETERMINING THE ENERGIES OF NUCLEI <br> FROM THE LATERAL DISTRIBUTION OF THE DENSITY OF THE SECONDARY-PARTICLE FLUX

The method due to Castagnoli [8] is based on the assumption that secondary pions originating from proton interactions are emitted isotropically in the c.m. frame. By virtue of Lorentz transformations, the mean value of $\ln \tan \theta_{i}$ in the laboratory frame $\left(\theta_{i}\right.$ is the emission angle of a secondary particle) is then proportional to the logarithm of the primary energy of the incident particle; that is, the lateral distribution of secondaries, which is usually analyzed in terms of the pseudorapidity $\eta=-\ln \tan (\theta / 2), d N / d \eta$, is sensitive, under certain conditions, to the primary energy. This method was applied in experiments where
nuclear emulsions and spark chambers were employed for detectors and where secondary photons from neutral-pion decays could not therefore be detected, which resulted in the violation of the condition of isotropy of charged-particle emission in the c.m. frame of colliding protons, since these secondary photons carry away an uncontrollable momentum fraction. This is not the whole story, however: in nucleonnucleus interactions, the left wing of the distribution $d N / d \eta$ is distorted by the contribution of slow particles produced in the subsequent interactions of the incident nucleon with target nucleons; this leads to the growth of fluctuations of $\langle\eta\rangle$ in individual events and, as a consequence, to an increase in the error in determining the energy. The aforementioned factors and experimental difficulties in detecting all slow particles traveling within the backward cone were the main reasons for a very large error in determining the energy by Castagnoli's method in individual events: $100-200 \%$ for energies in the range $0.1-$ 1 TeV . Methods for determining the energy that are based on computing the maximum value of $\eta$, which is also proportional to the logarithm of the primaryparticle energy, were developed in the RUNJOB experiment [6], whereby the effect of slow particles was eliminated [9]. For technical reasons, however, that experiment measured only secondary photons rather than charged particles.

Prior to demonstrating how the aforementioned problems were sidestepped, we will dwell upon general criteria for choosing the method. For a nuclear interaction, it is necessary to find a parameter $S$ (or a set of parameters) that can easily be measured with a specific array and which depends on the primaryparticle energy. Upon plotting the mean calibration dependence $\langle S\rangle(E)$, an energy value $E_{\text {meas }}$ can be associated with each individual event. The basic requirements are the following: first, the mean calibration dependence must be linear or must be close to a linear one, $\langle S\rangle \approx k E$ (if the energy dependence of $\langle S\rangle$ is much weaker than a linear dependence, small fluctuations in the measured parameter $S$ would lead to large errors in determining the energy); second, the error in determining the energy, $\delta\left(E_{\text {meas }} / E\right)$, must be independent of energy-otherwise, the measured spectra of particles would differ considerably from the true spectra [10].

Taking into account special features of the determination of energies from the emission angles of secondary particles and the aforementioned difficulties in this determination, we propose using a combined method that relies on a measurement of the emission angles both for charged and for the fastest neutral particles, on one hand, and which employs information about the energies of secondaries, on the other


Fig. 1. Layout of the proposed array ( $D 1$ and $D 2$ denote layers of coordinate-sensitive detectors).


Fig. 2. Schematic representation of the emission-angle distribution of secondary particles (solid curves) before and (dotted curves) after the converter for $1-$, $10-$, and $100-\mathrm{TeV}$ protons.
hand. The layout of the proposed array is displayed in Fig. 1.

The array in question consists of a target and a converter of photons in the form of a thin lead layer ( $h \sim 1-2 \mathrm{~cm}$ ) placed at a certain distance from the primary-interaction vertex ( $H \sim 20 \mathrm{~cm}$ ). A layer of coordinate-sensitive detectors that are capable of recording the number and the coordinates of charged particles (silicon microstrip detectors, which will be discussed below, can be used for such detectors) is proposed to be arranged underneath the converter.

A primary particle interacts in the target, where there arise secondary photons (from the decays of $\pi^{0}$ and $\eta$ mesons) and singly charged particles of energy $E^{i}$. The converter transforms almost all secondary photons incident on this layer of matter into a narrow electron beam owing to a cascade multiplication in lead. The number of product electrons is proportional to $\left(E^{i}\right)^{s}$ (where $s$ is the shower age, which is a function of the depth $h$ and the energy $E^{i}$; $s=0.1-0.2$ for $h=1-2 \mathrm{~cm})$. Some of the charged
particles incident on the converter interact within a thin lead layer, the multiplicity of secondaries from these interactions being logarithmically dependent on the energy $E^{i}$. As a result, the total number of singly charged particles (electrons, pions, kaons) below the converter, $N_{\text {after }}$, appears to be considerably greater than the number of particles before the converter, $N_{\text {before }}\left(N_{\text {after }} \sim N_{\text {before }} M\left(E^{i}, E, R\right)\right.$ ). The coefficient of multiplication, $M\left(E^{i}, E, R\right)$ proves to be dependent on the primary-particle energy $E$ and on the energies $E^{i}$ of secondary particles; it is also dependent on the distance $R$ from the shower axis, because the most energetic secondaries travel near the shower axis. The multiplication of particles is more intense at the center of a shower than at its periphery, with the result that the spatial density of particles changes upon traversing the converter. The mean value of $M$ increases from 3.5 at 100 GeV to 20 at 1000 TeV . Figure 2 shows schematically the variation of the spatial density of secondaries, $d N / d \eta$, where $\eta_{i}=-\ln \tan \left(\theta_{i} / 2\right)$. The fastest particles, which carry the bulk of the interaction energy and which, on the pseudorapidity scale, occur on the right wing of the distribution depicted in Fig. 2, have the largest coefficient of multiplication. The contribution of fast particles is emphasized by the converter, the shape of the distribution $d N / d \eta$ beginning to depend more sharply on energy.

The parameter $S$ characterizing the pseudorapidity distribution of the density of the secondary-particle flux was introduced as

$$
S\left(E_{0}\right)=\sum \eta_{i}^{2} N_{i}
$$

where $\eta_{i}=-\ln \tan \left(\theta_{i} / 2\right) \approx-\ln \left(r_{i} /(H / 2)\right)$; here, $r_{i}$ is the distance between the shower axis and the $i$ th coordinate-sensitive detector, which recorded $N_{i}$ particles, while $H$ is the distance between the plane of the coordinates of the coordinate-sensitive detectors and the particle-interaction vertex in the target. The shower axis is found by determining the maximum of the particle density. The parameter $S$ suppresses sizably the contribution of slow particles (owing to the fact that the function in question is quadratic in pseudorapidity) formed as the result of a cascade process in the target nucleus, thereby remedying one of the flaws in Castagnoli's method.

Alternatively, $S$ can be represented in the form

$$
S=\sum \eta_{i}^{2} N_{i}=\left\langle\eta^{2}\right\rangle N
$$

where $N$ is the total multiplicity and $\left\langle\eta^{2}\right\rangle$ is related, by definition, to the variance of the distribution as $\sigma_{\eta}^{2}=\left\langle\eta^{2}\right\rangle-\langle\eta\rangle^{2}$. The following features of the pseudorapidity distribution $d N / d \eta$ before the converter are known: the mean value $\langle\eta\rangle$ and the maximal value
$\eta_{\max }$ (which is proportional to the distribution width $\sigma_{\eta}$ ) grow logarithmically with increasing interaction energy [9]. The value of $S$ above the converter must then depend on energy rather weakly, as $\ln ^{3} E$. However, the converter changes considerably the shapes of the functions $\left\langle\eta^{2}\right\rangle(E)$ and $\langle N\rangle(E)$ and, as a consequence, the shape of $\langle S\rangle(E)$. For primary protons of energy in the range between 100 GeV and 1 PeV , Fig. 3 shows the parameter $S$ and the the total particle multiplicity $N$ versus energy before and after the converter. It can be seen that, after the converter, the parameter $S$ depends on the primary energy as a powerlaw function over the entire energy range under investigation, the exponent in this dependence being 0.7 to 0.8 . The errors in Fig. 3 represent the root-mean-square deviation in determining the energy of an individual event. As can be seen from Fig. 3 (and as will be demonstrated below), it is virtually independent of energy, amounting to about $60 \%$. Attempts at determining the energy by using only the energy dependence of the total multiplicity $N(E)$ yielded a poorer result-the error proved to be about $100 \%$.

It should be noted that the functions $\langle S\rangle(E)$ and $\langle N\rangle(E)$ are much more gently sloping before than after the converter-it is the application of the converter that radically improves the result. Therefore, the proposed method can be considered as a combination of the kinematical and the burst method.

The method can be used to determine the energies of nuclei. In doing this, it is necessary to take into account some special features of nucleus-nucleus collisions. In the interactions of an incident nucleus having an atomic number A and an energy $E_{A}$ with a target nucleus (carbon), only part of the nucleons of the incident nucleus, $N_{w}$, are involved in the interaction. The pseudorapidity distribution of secondary pions in the forward cone can be represented as the sum of the distributions for independent $p \mathrm{C}$ interactions at energy $E_{A} / A$ (in accordance with the superposition model). The parameter $S\left(E_{A} / A\right)$ for primary nuclei will then differ from $S(E)$ for primary protons at a fixed energy per nucleon only by the factor $N_{w}$; that is,

$$
S_{A}\left(E_{A} / A\right)=N_{w} S_{p}\left(E_{p}=E_{A} / A\right)
$$

However, part of the noninteracted nucleons of the incident nucleus $A, N_{w}$, will be emitted in the form of light fragments and spectator nucleons whose transverse momenta are much lower than those of secondary pions. The spectator nucleons make a significant contribution to the right wing in the pseudorapidity distribution, but they cause virtually no change in the energy dependence of $S$. It is more difficult to estimate the contribution of fragments that have not suffered interactions in the converter, since the response of a microstrip detector is proportional to the


Fig. 3. (a) Parameter $\langle S\rangle(E)=\sum \eta_{i}^{2} N_{i}$ and (b) total multiplicity $\langle N\rangle(E)$ versus the primary-proton energy (closed boxes) before and (closed circles) after the converter.
square of the charge of a particle that traverses the detector. These effects can be taken into a account through a Monte Carlo simulation. Relevant calculations will be described in the next section.

## 2. SIMULATION OF THE METHOD

The planned experiment was simulated with the aid of the GEANT package, which includes codes describing electromagnetic processes. Various generators were applied to treat hadron interactions. Originally, the FLUKA model was used a basic generator. Later on, this model was invoked only in describing hadron-nucleus interactions for energies below 50 GeV . High-energy hadron-nucleus and all kinds of nucleus-nucleus interactions were treated on the basis of the QGSJET model [11]. This code is tested by contrasting its predictions against collider data at laboratory energies up to about 500 TeV (the fragmentation region being excluded from this comparison) and is widely used in describing extensive atmospheric showers. It should be noted that QGSJET


Fig. 4. Calibration dependences $\langle S\rangle(E)$ for (open circles) primary protons, (open boxes) carbon nuclei, and (open triangles) iron nuclei at the converter-thickness values of (a) 1 and (b) 2 cm .
reproduces fairly well the experimental pseudorapidity distributions of secondaries produced by colliding proton and antiproton beams of energies up to $\sim 10^{15} \mathrm{eV}$, this energy value being rescaled to the laboratory frame.

The QGSJET generator includes the production of nuclear fragments having various masses. The transverse momenta of these fragments were generated according to an exponential distribution, as was proposed in [12]. This is consistent with modern model concepts and available experimental data. In performing our simulation, we traced the tracks of electrons and photons whose energies were in excess of 10 keV , the threshold for other particles being 60 keV . In the version of the calculations that is described here, we chose the following values for the parameters of the array: the thickness of the graphite target was 10 cm , the converter thickness was 1 to 2 cm of lead, and the air gap between the target and the converter was 20 cm . The calculation was performed for the cases where protons, C nuclei, and Fe nuclei were vertically incident on the target.

For a trigger, we took the requirement that more than four charged particles be produced between the target and the converter. The coordinates and the charges of particles at the upper plane of the converter and at the depths of 1 and 2 cm of lead were recorded in a database. In all, we obtained 15 groups, each containing, on average, 400 events: six groups of events for primary protons of energy ranging between $10^{11}$ and $10^{16} \mathrm{eV}$ (one group per one order of magnitude of energy and analogously for other primary-nucleus species), five groups of events for carbon nuclei of energy $10^{11}$ to $10^{15} \mathrm{eV} /$ nucleon, and four groups of events for iron nuclei of energy $10^{11}$ to $10^{14} \mathrm{eV} /$ nucleon.

As was indicated above, the parameter $S(E)=$ $\sum \eta_{i}^{2} N_{i}$ was proposed for determining the energy. In this section, we consider a more general form of it, $S(E)=\sum \eta_{i}^{k}$, where $k$ is varied from 1 to 4 and $\eta_{i}$ is the pseudorapidity of the $i$ th secondary particle at the level of detection; that is, we disregard the spatial resolution of detectors in analyzing the potential of the method. The calculations performed for various values of $k$ revealed that, in the energy range being considered, an optimum reconstruction of energy on the basis of the parameter $S$ for all types of nuclei simultaneously is accomplished at $k=2$, provided that the converter thickness is 1 or 2 cm of lead. The dependences $S(E)$ are displayed in Figs. $4 a$ and $4 b$ for the converter thicknesses of 1 and 2 cm , respectively.

The accuracy of energy reconstruction is determined by the fluctuations of the parameter $S$ used and by the slope of its dependence. For the powerlaw dependence $\langle S\rangle \sim E^{\beta}$, the relative error of the energy measurements is $\delta E=(1 / \beta) \delta S$. At the converter thickness of 2 cm , the slope exponent $\beta$ proved to be $0.80,0.77$, and 0.75 for incident protons, carbon nuclei, and iron nuclei, respectively. At the converter thickness of 1 cm , it appeared to be less by $0.1-$ 0.15 for all species of incident nuclei. The resulting errors in determining the energy, $\delta\left(E_{\text {meas }} / E\right)$, in an individual event are given in Table 1 for two versions of the converter. They are close, on average, to $60 \%$ for all nuclear species under investigation and are virtually independent of energy in the range $10^{11}-10^{16} \mathrm{eV}$ per particle. No pronounced difference of these values between the cases of $h=1$ and 2 cm has been revealed. The disintegration of heavy nuclear fragments in the converter upon their interaction with matter reduces fluctuations of the signal. In this respect, a converter of thickness $h=2 \mathrm{~cm}$ is preferable. For $h>$ 2 cm , however, the rate of photon multiplication in lead is much greater than the rate of charged-particle multiplication, in which case fluctuations associated with masking the contribution of charged particles may increase. In an actual array, we propose using a

Table 1. Errors in determining the energy, $\delta\left(E_{\text {meas }} / E\right)$, versus the type of the primary nucleus, the energy, and the converter thickness $h$

| $h, \mathrm{~cm}$ | Energy of the primary nucleus, eV/nucleon |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $10^{11}$ |  |  |  |  |  |
|  | $10^{12}$ | $10^{13}$ | $10^{14}$ | $10^{15}$ | $10^{16}$ |  |  |
| 1 | $p$ | 0.67 | 0.70 | 0.63 | 0.57 | 0.57 | 0.61 |
| 1 | C | 0.73 | 0.68 | 0.71 | 0.72 |  |  |
| 2 | Fe | 0.51 | 0.40 | 0.52 | 0.62 |  |  |
| 2 | $p$ | 0.72 | 0.69 | 0.61 | 0.55 | 0.56 | 0.60 |
| 2 | C | 0.69 | 0.70 | 0.67 | 0.69 |  |  |

converter of thickness 1 cm ; for particles arriving at large angles, the effective thickness will then be about 2 cm .

An example of the distribution of events with respect to the reconstructed energy, $W\left(\log \left(E_{\text {meas }} / E\right)\right)$, is given in Figs. $5 a$ and $5 b$ for primary protons and iron nuclei.

A feature peculiar to the method is that the distributions in question exhibit a pronounced asymmetry. A significant contribution to the fluctuations comes from the tail in the region of underestimated values of $E_{\text {meas }}$. The distributions displayed in Fig. 5 were obtained at a fixed primary energy; that is, they represent the probability $W\left(E, E_{\text {meas }}\right)$ of assigning a particle of energy $E$ the energy $E_{\text {meas }}$. In this case, the mean error in determining the energy on a logarithmic scale is $\delta\left(\log \left(E_{\text {meas }} / E\right)\right)=0.46,0.49$, and 0.54 for


Fig. 5. Distribution of events with respect to $\log \left(E_{\text {meas }} / E\right)$ for (a) primary protons and (b) iron nuclei at a converter thickness of 2 cm . The closed and the open circles represent, respectively, the direct distribution function $W$ and the inverse distribution function $W^{*}$ that takes into account the a priori spectrum of particles.
protons, carbon nuclei, and iron nuclei, respectively. In measuring power-law spectra of particles, the inverse distribution function $W^{*}\left(E_{\text {meas }}, E\right)$ defined as the probability that, at a fixed measured energy $E_{\text {meas }}$, the true particle energy is $E$ is of importance. According to the Bayes theorem, the direct and the inverse distribution function are related by the equation

$$
\begin{gathered}
W^{*}\left(E_{\text {meas }}, E\right) \\
=W\left(E, E_{\text {meas }}\right) F(E) / \int W\left(E, E_{\text {meas }}\right) F(E) d E
\end{gathered}
$$

where $F(E)$ is the a priori spectrum of hadrons. If this a priori spectrum has a power-law form, $F(E)=$ $E^{-\gamma}$, the contribution of small values of $E_{\text {meas }} / E$ is suppressed in proportion to $\left(E_{\text {meas }} / E\right)^{\gamma-1}$. The inverse distribution function $W^{*}$ is represented by the dotted curve in Fig. 5. It is much narrower than the direct distribution function $W$. The calculation by the above formula at $\gamma=2.7$ yields $\delta\left(\log \left(E_{\text {meas }} / E\right)\right)=$ $0.22,0.23$, and 0.25 for protons, carbon nuclei, and iron nuclei, respectively.

In measuring monotonic power-law spectra of particles, the absolute error is not very importantthe energy independence of the errors is quite sufficient [10]. In the case of uniform distribution functions depending only on the ratio $E_{\text {meas }} / E$, the measured spectrum is related to the true spectrum by the equation $F\left(E_{\text {meas }}\right)=\left\langle\left(E_{\text {meas }} / E\right)^{\gamma-1}\right\rangle F(E)$ [10]. If $\left\langle E_{\text {meas }} / E\right\rangle \sim 1$, the intensity of the measured spectrum is always higher than the intensity of the true spectrum.

A small error in determining the energy is necessary if some structures are presumed in the measured spectrum. In order to demonstrate measurements of a peak in the particle spectrum and of the knee region in the spectrum, the true particle spectra and the spectra that are measured by our method are presented in Figs. $6 a$ and $6 b$ with these features. For the sake of visual convenience, the spectra are multiplied by

Table 2. Errors in determining the energy, $\delta\left(E_{\text {meas }} / E\right)$, versus the type and the energy of the primary nucleus with allowance for the process of detection by strip detectors

| Type <br> of <br> nucleus | Energy of the primary nucleus, eV/nucleon |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $10^{11}$ | $10^{12}$ | $10^{13}$ | $10^{14}$ | $10^{15}$ | $10^{16}$ |
| $p$ | 0.77 | 0.71 | 0.62 | 0.62 | 0.54 | 0.54 |
| C | 0.75 | 0.69 | 0.72 | 0.78 |  |  |
| Fe | 0.45 | 0.77 | 0.78 | 0.81 |  |  |

$E^{2.7}$. As can be seen from Fig. 6, the structures of the spectra are reconstructed rather well. That the intensity of the measured spectrum of particles is higher by the factor $\left\langle\left(E_{\text {meas }} / E\right)^{\gamma-1}\right\rangle$ leads to a shift of the knee region and of the peak region-these effects, which are associated with the presence of fluctua-


Fig. 6. Presumed spectra of primary cosmic rays with (a) a peak and ( $b$ ) a knee (curves passing through closed circles) along with corresponding particle spectra measured by the method proposed here (curves passing through closed boxes).
tions, can easily be taken into account in analyzing experimental data.

## 3. POTENTIAL OF THE METHOD WITH ALLOWANCE FOR DETECTION PROCESSES

The above results refer to the case of an ideal instrument that can measure the coordinates of all secondary particles to as high a precision as is desired. There is, however, the question of whether the method is applicable in the case of actual instruments, where the detection procedure has a finite spatial resolution. The first question to be answered here is that of how the calibration dependence and the error in determining the energy change in this case. In order to avoid technical details, we performed a calculation for the case where the lateral distribution of secondaries is roughened to a considerable extent. We assumed that, under the converter, there are two layers of coordinate-sensitive detectors; that these layers are oriented orthogonally to each other in space; and that each of these consists of strips that have a thickness of $50 \mu \mathrm{~m}$ and a length equal to that of the entire array. The signal is read off each strip. Thus, the total ionization is fixed in each strip (as a matter of fact, it determines the number of secondaries that fall within this strip); that is, the lateral distribution of secondaries is integrated with respect to $x$ and $y$. On average, the distributions of secondaries with respect to $x$ and $y$ are symmetric; therefore, two detector layers yield two independent measurements of the transverse density, whereby the accuracy in determining the energy is improved.

For the case being considered, the parameter $S$ was modified: instead of the emission angle of


Fig. 7. Dependence $\left\langle S_{2}\right\rangle(E)$ for primary (open circles) protons, (open boxes) carbon nuclei, and (open triangles) iron nuclei that was obtained with allowance for the process of detection by coordinate-sensitive detectors.
a secondary particle, we took its projection onto the observation plane. Since $\eta_{i}=-\ln \left(\tan \left(\theta_{i} / 2\right)\right)=$ $\ln \left(2 H / r_{i}\right)$, where $H$ is the distance from the primaryinteraction vertex to the detector plane and $r_{i}$ is the distance between the secondary particle and the shower axis, the quantities $\phi_{i}^{x}=\ln \left(2 H / x_{i}\right)$ and $\phi_{i}^{y}=$ $\ln \left(2 H / y_{i}\right)$, where $x_{i}$ and $y_{i}$ stand for the distance from the strip center to the shower axis along the respective coordinate, were chosen for new variables. For the analog of $S$, we can then take the parameter

$$
S_{2}=f r a c 12\left(\sum \phi_{i}^{x} N_{i}+\phi_{i}^{y} N_{i}\right)
$$

where $N_{i}^{x}$ and $N_{i}^{y}$ represent the number of particles that hit the strip (for each coordinate axis, the shower axis is found as the line that breaks down the number of particles into two equal parts). For the coordinate, we used the position of the midpoint of the relevant strip.

It turned out that the modified parameter $S_{2}$ averaged over the coordinates $x$ and $y$ is also a powerlaw function of energy, $\left\langle S_{2}\right\rangle(E) \sim E^{\beta}$. It is displayed in Fig. 7 for various types of primary nuclei. The exponents $\beta$ in this power-law dependence proved to be very close ( $\beta=0.78$ for protons, $\beta=0.79$ for carbon nuclei, and $\beta=0.71$ for iron nuclei) to those obtained previously for the case where the coordinates of each secondary are recorded ( $\beta=0.80$ for protons, $\beta=0.77$ for carbon nuclei, and $\beta=0.75$ for iron nuclei). The direct distributions with respect to the energy reconstructed with the aid of the parameter $S_{2}, W\left(E_{\text {meas }} / E\right)$, also differ insignificantly from the distributions in Fig. 5, which were obtained without taking into account processes of detection by coordinate-sensitive detectors. A characteristic tail in the region of underestimated energies is present in this case as well, but, as was shown in the preceding section, it has only a modest effect on the actual accuracy of the method. The resulting values of the errors in determining the energy through the parameter $S_{2}$ are quoted in Table 2. On the logarithmic scale, they are (for the case where the exponent of the a priori spectrum is $\gamma=2.7) \delta\left(\log \left(E_{\text {meas }} / E\right)\right)=0.22$, 0.219 , and 0.265 for protons, carbon nuclei, and iron nuclei. This indicates that, within the method being discussed the most pronounced fluctuations are associated with the physical fluctuations of the production of secondary particles in a nuclear interaction rather than with the method of detection.

The effect that errors introduced by microstrip detectors exert on the accuracy in determining the primary-particle energy was investigated here by using part of the statistics presented in [13]. It turned out that the calibration dependences $\left\langle S_{2}\right\rangle(E)$ and the errors in determining the energy have undergone virtually no changes. This can easily be understood by
comparing the errors of the measurements with fluctuations of the multiplicities of secondary particles. The fraction of nonrelativistic particles after 2 cm of lead is still very small, so that the fluctuations that they introduce in the total ionization are insignificant. The fluctuations of the ionization for relativistic secondary particles can be estimated at 10 to $15 \%$. Fluctuations that are introduced by electronics and detector noises are on the same order of magnitude. Therefore, the total contribution of all fluctuations of measurements does not exceed $20 \%$; that is, it is negligibly small in relation to multiplicity fluctuations that are greater than $100 \%$ per strip.

## 4. APPLICABILITY RANGE OF THE METHOD

The proposed method for determining the primaryparticle energy and the possible design of the respective array possess a fairly high potential for studying primary cosmic radiation in space-vehicle-borne experiments. Such an implementation of this procedure could solve many topical problems of astrophysics that have hitherto defied any attempt at tackling them by means of modern technologies. The dependence of a geometric factor on the weight of equipment that we have described is much more favorable than that in burst detectors of similar energy resolution. By way of example, we indicate that (see [14]) an array of weight 500 kg can have a geometric factor of about 3, whereas a burst detector of the same weight has a geometric factor that is approximately ten times smaller than that. With respect to the weight-aperture-dimension relationship, the equipment constructed on the basis of the ideas developed here would possess unique properties-none of the facilities known to date would be able to compete with it in what is concerned with detecting cosmic rays of energy in excess of $10^{12} \mathrm{eV}$. Moreover, the structure of the proposed equipment may admit its design in the form of separate modules; that is, one could construct a basic module of dimensions, say, $30 \times 30 \times 30 \mathrm{~cm}^{3}$ and weight about 40 to 50 kg and take this module beyond the atmosphere, whereupon the experiment in question would begin. Further, advancements toward higher energies are accomplished along with a gradual increase in the number of such modules in the orbit used. This principle of designing equipment would make it possible to take into account, to a maximum possible degree, the structural features of the space vehicle used and to facilitate the implementation of the respective cosmic experiments as a whole.

To conclude this section, we address the question of what the detectors of the proposed array would record if multiparticle-production events undergo an
abrupt change in the energy range $10^{15}-10^{16} \mathrm{eV}$ ( this hypothesis, which was put forth by S.I. Nikolsky, has been discussed for many years in cosmic-ray physics as the possible explanation of the knee in the spectrum of extensive air showers with respect to the number of electrons). As was suggested in [15], the emergence of a considerable number (about $50 \%$ ) of proton interactions in which the multiplicity of charged secondary pions is $10^{2}$ to $10^{3}$ times greater than the mean multiplicities predicted by currently available models may be one of the possible scenarios of the above changes in multiparticle-production events. It is foreseen that the charge of a primary particle would be determined, to a very high precision, by silicon detectors positioned at the upper plane of the target ( see Fig. 1). In relation to what is observed for heavy nuclei, which also generate events characterized by a very high multiplicity, the lateral distribution of low-energy secondary pions produced in proton interactions must be much narrower because of the difference in the energy per nucleon. This class of high-multiplicity events generated by a primary particle of small charge is easily identifiable. If an additional plane of strip detectors is arranged above the converter, the fraction of neutral pions produced in an anomalous nuclear-interaction event can be assessed on the basis of the relation between the multiplicity of secondaries above the converter and the multiplicity of secondaries below it.

## CONCLUSION

The proposed method for determining the energy of particles of primary cosmic radiation on the basis of the lateral distribution of the secondary-particleflux density makes it possible to construct arrays of large area and high sensitivity at a comparatively small weight of the array. The method is applicable to all nuclei of primary cosmic rays over a wide energy range (from $10^{11}$ to $10^{16} \mathrm{eV}$ per particle). A fairly small error in determining the energy in an individual event $\left[\delta\left(\log \left(E_{\text {meas }} / E\right)\right) \sim 0.2-0.25\right.$ for a measurement of power-law spectra of primary cosmic rays with a slope exponent of $\gamma \sim 2.5-3.3$ ] enables one to resolve some features of the spectra of primary cosmic rays (such as the presence of a knee in the spectrum and the existence of peaks). The proposed
design of the detector will make it possible to single out the class of events that are characterized by an anomalously high multiplicity.

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